The Origin of Risk

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What drives individual risk-taking decisions and how do they affect aggregate risk?

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- Since TFP multiplies the input bundle, larger firms manage risk more aggressively

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- Larger firms and those with low markups are less volatile and covary less with GDP
- We find support for these predictions in detailed firm-level Spanish data

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- Because of endogenous risk, distortions can make GDP more volatile

We calibrate the model to the Spanish economy

- Removing distortions lead to a large decline in aggregate volatility

Literature review

Most of macroeconomics takes risk as exogenous (at the micro and/or macro level)

- In models with individual firms, firm-level risk is generally exogenous but macro risk can be endogenous
  - Khan and Thomas (2008), Clementi and Palazzo (2016), Bloom et al. (2018), and many others
- In endogenous growth models, firms influence the growth rate of TFP but not its variance
  - Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Jones (1995)
- Corporate finance literature where managers influence how risky a project is
  - Jensen and Meckling (1976), Ross (1977)
- Wedges in production network economies
  - Jones (2011), Baqaee and Farhi (2019), Liu (2019) and Bigio and La'O (2020)
- Technique choice in production networks
  - Oberfield (2018), Acemoglu and Azar (2020), Kopytov et al. (2024)

A model of endogenous risk

Static model with two types of agents

1. A representative household owns the firms, supplies labor and risk management resources
2. N firms produce differentiated goods using labor and intermediate inputs
   - Firm i has constant returns to scale Cobb-Douglas production function

▸ ζ

$$F(\delta_i, L_i, X_i) = e^{a_i(\varepsilon, \delta_i)} \zeta_i L_i^{1-\sum_{j=1}^{N} \alpha_{ij}} \prod_{j=1}^{N} X_{ij}^{\alpha_{ij}}$$

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- Examples: a river floods, discovery of a new drug, war between two countries, epidemic, etc.
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$$a_i(\varepsilon, \delta_i) = \delta_i^\top \varepsilon$$

Managing risk (picking δ_i) requires risk management resources R_i supplied by the household

$$R_i = \kappa_i(\delta_i) = \frac{1}{2}(\delta_i - \delta_i^\circ)^\top H_i (\delta_i - \delta_i^\circ)$$

where δ_i° is the *natural* risk exposure ($R_i = 0$), and H_i is a positive definite matrix

Representative household

Owns the firms, supplies one unit of labor inelastically, supplies risk management resources

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Values the consumption bundle (GDP)

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Maximizes King, Plosser, Rebelo (1988) preferences

$$\mathcal{U}(Y)\,\mathcal{V}(R)$$

where $\mathcal{U}$ is CRRA with risk aversion $\rho \geq 1$, and disutility of risk management $\mathcal{V}(R)$ is ▸ Details

$$\mathcal{V}(R) = \exp\left(-\eta\left(1-\rho\right)R\right)$$

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Budget constraint in each state of the world (set $W_L = 1$ from now on)

$$\sum_{i=1}^{N} P_i C_i \leq W_L + W_R R + \Pi$$

Timing and distortions

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- Example: markups, taxes, or other distortions

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Cobb-Douglas unit cost is

$$K_i (\delta_i, P) = \frac{1}{e^{a_i(\varepsilon, \delta_i)}} \prod_{j=1}^{N} P_j^{\alpha_{ij}}$$

Risk-taking decision

Firm choose their risk exposure to maximize expected discounted profits

$$\delta_i^* \in \arg\max_{\delta_i \in \mathcal{A}_i} \mathrm{E}\left[\Lambda\left[P_i Q_i - K_i\left(\delta_i, P\right) Q_i - \kappa_i\left(\delta_i\right) W_R\right]\right]$$

where Q_i is *equilibrium* demand and Λ is the stochastic discount factor of the household.

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Firms prefer risk exposures with

1. low risk management expenses $\kappa_i\left(\delta\right)$
2. high expected TFP (low expected unit costs K_i)
3. low covariance with GDP

Equilibrium definition

An *equilibrium* is a risk choice for every firm δ^* and a stochastic tuple $(P^*, W_R^*, C^*, L^*, R^*, X^*, Q^*)$ such that

1. (Optimal technique choice) For each i, factor demand L_i^*, X_i^* and R_i^*, and the risk exposure decision δ_i^* solves the firm's problem.
2. (Consumer maximization) The consumption vector C^* and the supply of risk managers R^* solve the household problem.
3. (Unit cost pricing) For each i, $P_i = (1+\tau_i)\, K_i\,(\delta_i, P)$.
4. (Market clearing) For each i,

$$C_i^* + \sum_{j=1}^{N} X_{ji}^* = Q_i^* = F_i\left(\alpha_i^*, L_i^*, X_i^*\right), \quad \sum_{i=1}^{N} L_i^* = 1, \text{ and } \sum_{i=1}^{N} \kappa_i\left(\delta_i^*\right) = R^*.$$

Two measures of supplier importance

Cost-based Domar weight:

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- Depends on demand from household (β) and other firms ($\mathcal{L} = (I - \alpha)^{-1} = I + \alpha + \alpha^2 + \ldots$)
- Captures firm's importance as a supplier (share of production costs)

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Revenue-based Domar weight:

$$\omega^{\top} = \beta^{\top}\mathcal{L} = \beta^{\top}\left(I - [\text{diag}\,(1 + \tau)]^{-1}\alpha\right)^{-1}$$

- Also captures importance as a supplier (share of revenues)
- Declines with wedges τ

Define aggregate risk exposure Δ as

$$\Delta := \delta^\top \tilde{\omega}$$

- Firms with high cost-based Domar weights contribute more to aggregate risk exposure

Determinants of GDP

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Lemma

$$\log Y = y = \Delta^{\top}\varepsilon - \tilde{\omega}^{\top}\log(1+\tau) - \log(\text{Labor share}(\omega,\tau))$$

- Without distortions ($\tau = 0$) we have Hulten's theorem: $y = \Delta^{\top}\varepsilon = \omega^{\top}a(\varepsilon,\delta)$

Aggregate risk

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Impact of Σ

- A marginal increase in Σ_{mm} raises $\mathbb{V}[y]$ by Δ_m^2
  - Both $\Delta_m \gg 0$ and $\Delta_m \ll 0$ are bad for $\mathbb{V}[y]$
- If the economy is positively exposed to m and n, increasing Σ_{mn} raises $\mathbb{V}[y]$.
- If $\Delta_m > 0$ and $\Delta_n < 0$, the shocks offset each other. Higher Σ_{mn} reduces $\mathbb{V}[y]$.

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Impact of Δ

$$\frac{d\,\mathrm{V}[y]}{d\Delta_m} = 2\,\mathrm{Cov}[y, \varepsilon_m] = 2\sum_n \Delta_n \,\mathrm{Cov}[\varepsilon_n, \varepsilon_m]$$

- Extra exposure to ε_m increases volatility if ε_m is positively correlated with GDP

Firm risk-taking decision

Lemma

The equilibrium risk exposure decision δ_i solves

$$\mathcal{E} \underbrace{K_i Q_i}_{\substack{\text{cost of} \\ \text{goods sold}}} = \underbrace{W_R \nabla \kappa_i \left(\delta_i\right)}_{\substack{\text{marginal cost} \\ \text{of exposure}}},$$

where $\mathcal{E}$ is the value of exposure, given by $\mathcal{E} := \mathrm{E}\left[\varepsilon\right] + \mathrm{Cov}\left[\lambda, \varepsilon\right]$.

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Benefit of increasing δ_i grows with the size of the firm since TFP multiplies the input bundle

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Equation for $\mathcal{E}$ implies that firms prefer risk factors with

- high expected value $\mu = \mathrm{E}[\varepsilon]$ and negative covariance with GDP ($\mathrm{Cov}[\lambda, \varepsilon] > 0$)
- Risk factor is “good” if $\mathcal{E} > 0$ and “bad” if $\mathcal{E} < 0$

Existence, uniqueness and efficiency

Planner's problem

Define $\bar{\kappa}_{SP}(\Delta)$ as the smallest risk management utility cost needed to achieve Δ.

$$\bar{\kappa}_{SP}(\Delta) := \min_{\delta} -\log V\left(\sum_{i=1}^{N} \kappa_i(\delta_i)\right), \quad \text{subject to } \Delta = \delta^\top \tilde{\omega}$$

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$$\mathcal{W}_{SP} := \max_{\Delta} \underbrace{\Delta^\top \mu}_{\mathrm{E}[y_{SP}]} - \frac{1}{2}(\rho - 1)\underbrace{\Delta^\top \Sigma \Delta}_{\mathrm{V}[y_{SP}]} - \bar{\kappa}_{SP}(\Delta)$$

The planner prefers aggregate risk exposure vectors Δ with

- high expected GDP $\mathrm{E}[y_{SP}]$
- low GDP volatility $\mathrm{V}[y_{SP}]$
- low risk management cost $\bar{\kappa}_{SP}$

Equilibrium characterization through fictitious planner

Define $\bar{\kappa}(\Delta)$ as the perceived smallest risk management utility cost needed to achieve Δ.

$$\bar{\kappa}(\Delta) := \min_{\delta} - \log V\left(\sum_{i=1}^{N} g_i \kappa_i(\delta_i)\right), \quad \text{subject to } \Delta = \delta^\top \tilde{\omega}$$

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Proposition (fictitious planner's problem)

There exists a unique equilibrium, and it solves

$$\mathcal{W}_{dist} := \max_{\Delta} \underbrace{\Delta^\top \mu - \tilde{\omega}^\top \log(1+\tau) - \log \Gamma_L}_{\mathrm{E}[y]} - \frac{1}{2}(\rho - 1)\underbrace{\Delta^\top \Sigma \Delta}_{\mathrm{V}[y]} - \bar{\kappa}(\Delta).$$

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The equilibrium solves a distorted planning problem

- Still seeks to maximize $\mathrm{E}[y]$ and minimize $\mathrm{V}[y]$
- But distorted perception of the cost of managing risk ($\bar{\kappa}$ instead of $\bar{\kappa}_{SP}$)

Determinants of equilibrium risk

Equilibrium risk exposure

First-order of fictitious planning problem

$$\underbrace{\mathcal{E}(\Delta)}_{\substack{\text{marginal}\\ \text{benefit of } \Delta}} = \underbrace{\nabla\bar{\kappa}(\Delta)}_{\substack{\text{marginal}\\ \text{cost of } \Delta}}$$

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Proposition

Let γ be either μ_m or Σ_{mn}. Then

$$\frac{d\Delta}{d\gamma} = \mathcal{H}^{-1} \frac{\partial \mathcal{E}}{\partial \gamma},$$

where $\mathcal{H}^{-1} := \left(\nabla^2 \bar{\kappa} + (\rho - 1)\Sigma\right)^{-1}$ is an $M \times M$ positive definite matrix.

- The vector $\partial \mathcal{E} / \partial \gamma$ captures the *direct* impact of γ on the attractiveness of risk factors ▸ $\partial \mathcal{E} / \partial \gamma$
- The matrix $\mathcal{H}^{-1}$ propagates that impact to exposure vector Δ

Equilibrium risk exposure

Corollary

1. An increase in μ_m raises Δ_m
2. An increase in Σ_{mm} reduces Δ_m if $\Delta_m > 0$ and increases Δ_m if $\Delta_m < 0$

- A marginal increase in Σ_{mm} raises $\mathrm{V}[y]$ by Δ_m^2 $\rightarrow$ When Σ_{mm} increases we want to reduce Δ_m^2

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What is the impact of μ_m or Σ_{mm} on Δ_n with $m \neq n$? Off-diagonal terms of $\mathcal{H}^{-1}$ are important.

- If $\left[\mathcal{H}^{-1}\right]_{mn} > 0$, m and n are *global complements* $\rightarrow$ an increase in $\mathcal{E}_m$ increases in Δ_n
- If $\left[\mathcal{H}^{-1}\right]_{mn} < 0$, m and n are *global substitutes* $\rightarrow$ an increase in $\mathcal{E}_m$ decreases Δ_n

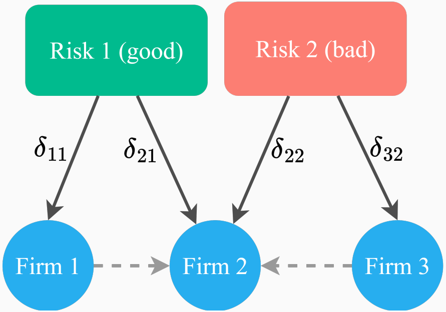
Example of substitution patterns

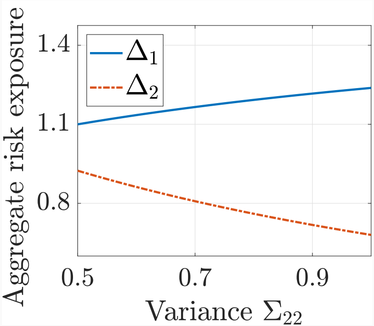
There are two regions both with their specific shocks

- Region 1: more productive in expectation (Risk 1 – good risk)
- Region 2: bigger shocks (Risk 2 – bad risk)

Firm 2 must decide where to locate plants

- Challenging to manage plants in different locations → risks are substitutes





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general case

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Corollary

In a diagonal economy, a higher wedge τ_i

1. increases Δ_m for all m such that $\mathcal{E}_m < 0$ (bad risks)
2. reduces Δ_m for all m such that $\mathcal{E}_m > 0$ (good risks)

- Higher wedges make firms shrink $\rightarrow$ manage risk less aggressively

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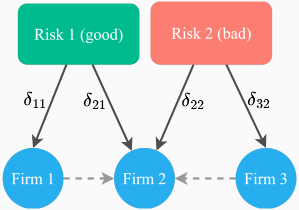
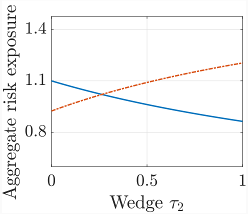




(Blue: good risk; Red: bad risk)

Implications for GDP and Welfare

Implications for GDP

general case

In general, our endogenous risk mechanism is important for understanding the aggregate impact of changing risk.

Use ∂ to denote changes in the economy with exogenous risk

Proposition

In a diagonal economy:

$$\operatorname{sign}\left(\frac{d\,\mathrm{E}[y]}{d\mu_m} - \frac{\partial\,\mathrm{E}[y]}{\partial\mu_m}\right) = \operatorname{sign}(\mu_m) \quad \text{and} \quad \frac{d\,\mathrm{V}[y]}{d\Sigma_{mm}} - \frac{\partial\,\mathrm{V}[y]}{\partial\Sigma_{mm}} < 0.$$

- Increasing μ_m raises Δ_m $\rightarrow$ additional increase in $\mathrm{E}[y]$ if $\mu_m > 0$ compared to fixed risk
- Increasing Σ_{mm} decreases $|\Delta|$ $\rightarrow$ smaller increase in $\mathrm{V}[y]$ than with fixed risk

Distortions can increase aggregate volatility

Proposition (single risk factor)

$$\text{sign}\left(\frac{d\,\mathrm{E}[y]}{d\tau_i} - \frac{\partial\,\mathrm{E}[y]}{\partial\tau_i}\right) = -\text{sign}(\mu\mathcal{E}) \qquad \text{and} \qquad \text{sign}\left(\frac{d\,\mathrm{V}[y]}{d\tau_i} - \frac{\partial\,\mathrm{V}[y]}{\partial\tau_i}\right) = -\text{sign}(\Delta\mathcal{E}).$$

Suppose $\mathcal{E} < 0$ (bad risk, e.g. business cycle): increasing τ_i makes firms more exposed to risk factor

- if $\mu < 0$ this leads to a decline in $\mathrm{E}[y]$
- if $\Delta > 0$ the economy becomes even more exposed and $\mathrm{V}[y]$ increases
- Inefficient allocation of risk exposure: a new channel that τ_i could lead to welfare losses.

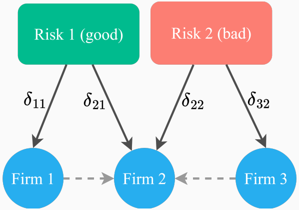
Distortions can increase aggregate volatility

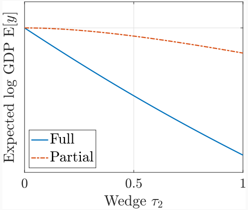
Proposition (single risk factor)

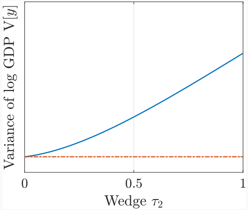
$$\text{sign}\left(\frac{d\,\mathrm{E}[y]}{d\tau_i} - \frac{\partial\,\mathrm{E}[y]}{\partial\tau_i}\right) = -\text{sign}(\mu\mathcal{E}) \quad \text{and} \quad \text{sign}\left(\frac{d\,\mathrm{V}[y]}{d\tau_i} - \frac{\partial\,\mathrm{V}[y]}{\partial\tau_i}\right) = -\text{sign}(\Delta\mathcal{E}).$$

Suppose $\mathcal{E} < 0$ (bad risk, e.g. business cycle): increasing τ_i makes firms more exposed to risk factor

- if $\mu < 0$ this leads to a decline in $\mathrm{E}[y]$
- if $\Delta > 0$ the economy becomes even more exposed and $\mathrm{V}[y]$ increases
- Inefficient allocation of risk exposure: a new channel that τ_i could lead to welfare losses.







Reduced-form evidence

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Model: firms with large Domar weights and small markups are less volatile and less corr. with GDP

▸ Details

Reduced-form evidence

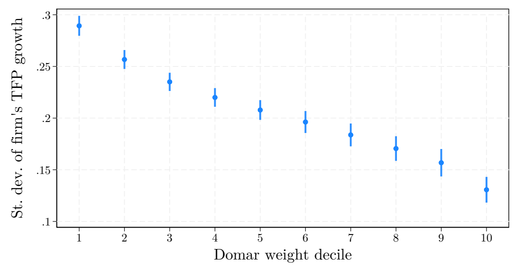
Model: firms with large Domar weights and small markups are less volatile and less corr. with GDP

▸ Details

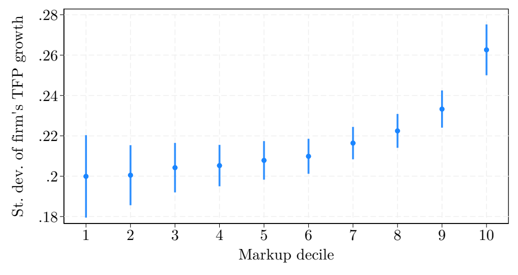
We test these predictions in the data

- Use detailed micro data from the near-universe of firms in Spain between 1995 and 2018 (Orbis) (7,513,081 firm-year observations)
- Compute markups using control function approach (De Loecker and Warzynski, 2012) ▸ Details
- Back out TFP growth as a residual

TFP growth volatility



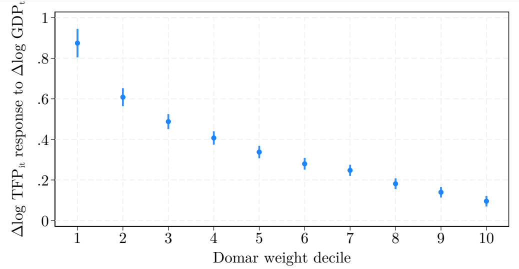
(a) TFP volatility by Domar weight decile



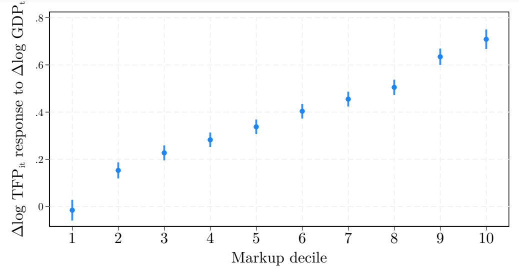
(b) TFP volatility by markup decile

▸ Details

Covariance of TFP growth with GDP growth



(c) Sensitivity of firm TFP to GDP by Domar weight decile



(d) Sensitivity of firm TFP to GDP by markup decile

▸ Details

Calibration

Mapping our model to the data

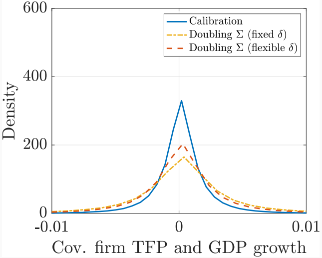
- Sector-level I-O linkages + Firm-level Heterogeneity in productivity and markup
- We aim at replicating as much of the firm-level Spanish data as possible
- Our calibrated model has 62 sectors and 492,917 individual firms
- We invert parts of the model to exactly match some moments
    1. Sectoral consumption shares and input/output cost shares
    2. Firm shares in sectoral sales
    3. Variance of firm TFP growth
    4. Covariance of firm TFP growth and GDP growth
    5. Variance of GDP growth

▸ Details

Doubling Σ

What if we double the volatility Σ of the risk factor?

	Calibration	Doubling Σ	
		Fixed δ	Flexible δ
Agg. risk exposure Δ	0.014	0.014	0.011
Exposure value $\mathcal{E}$	-0.06	-0.11	-0.09
Std. Dev. of GDP growth	2.4%	3.1%	2.6%



- Fixed δ: Large increase in GDP variance; exposure to ε_t becomes more harmful ($\mathcal{E}$ declines)
- Flexible δ: Firms manage risk more aggressively which limits increase in $\mathbb{V}[y]$

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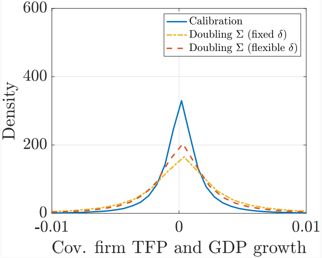

- Fixed δ: Large increase in GDP variance; exposure to ε_t becomes more harmful ($\mathcal{E}$ declines)
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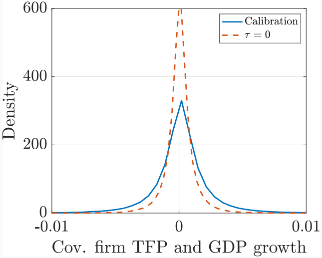
Impact of risk can be overestimated if reaction of agents is not taken into account

Removing distortions

What if we set wedges τ to zero?

	Calibration	No wedges	
		Fixed δ	Flexible δ
Agg. risk exposure Δ	0.014	0.014	0.007
Exposure value $\mathcal{E}$	-0.06	-0.06	-0.03
Std. Dev. of GDP growth	2.4%	2.4%	1.7%



- Fixed δ: Since only impact of τ is through δ, there is no change.
- Flexible δ: Firms manage risk more aggressively so $\mathbb{V}[y]$ declines

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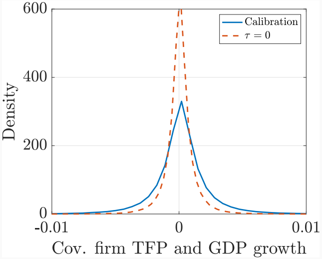


- Fixed δ: Since only impact of τ is through δ, there is no change.
- Flexible δ: Firms manage risk more aggressively so $\mathbb{V}[y]$ declines

Distortions can make GDP more volatile

Conclusion

Conclusion

Main contributions

- We construct a model of endogenous risk, at both the micro and macro levels.
- Model predicts which firms are more volatile and covary more with business cycle.
- Distortions lead to less aggressive risk management and can increase GDP volatility.

Future research

- What if there are entrepreneurs who cannot diversify their risk?
- Mechanisms would interact with capital/investment. Fully dynamic business cycle model.

Expression for $\zeta(\alpha_i)$

The function $\zeta(\alpha_i)$ is

$$\zeta(\alpha_i) = \left[\left(1 - \sum_{j=1}^{n} \alpha_{ij} \right)^{1 - \sum_{j=1}^{n} \alpha_{ij}} \prod_{j=1}^{n} \alpha_{ij}^{\alpha_{ij}} \right]^{-1}$$

This functional form allows for a simple expression for the unit cost K

Back

Risk aversion and ρ

Given the log-normal nature of uncertainty $\rho \lesseqgtr 1$ determines whether the agent is risk-averse or not. To see this, note that when $\log C$ normally distributed, maximizing

$$\mathrm{E}\left[C^{1-\rho}\right]$$

amounts to maximizing

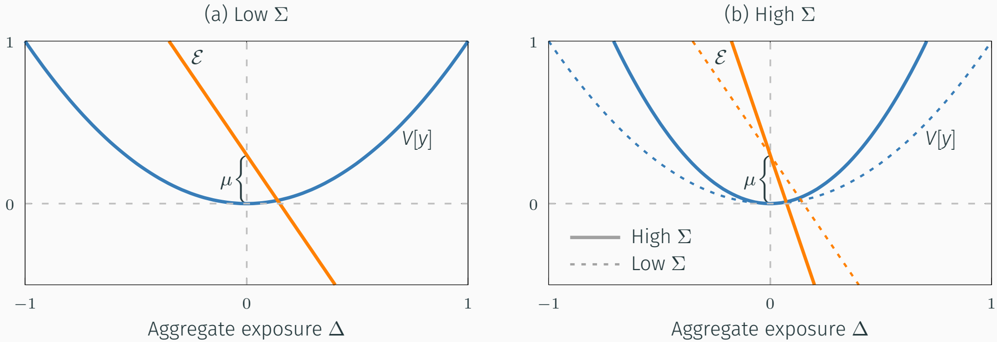
$$\mathrm{E}\left[\log C\right] - \frac{1}{2}\left(\rho - 1\right)\mathrm{V}\left[\log C\right].$$

◂ Back

Expressions for $\partial\mathcal{E}/\partial\gamma$

The direct impact of changes in (μ, Σ) is given by

$$\frac{\partial\mathcal{E}}{\partial\mu_m} = \mathbf{1}_m \qquad \text{and} \qquad \frac{\partial\mathcal{E}}{\partial\Sigma_{mn}} = -\frac{1}{2}(\rho - 1)\left(\Delta_m\mathbf{1}_n + \Delta_n\mathbf{1}_m\right).$$



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Impact of wedges

Proposition

The response of the equilibrium aggregate risk exposure Δ to a change in wedge τ_i is given by

$$\frac{d\Delta}{d\tau_i} = \mathcal{T} \left(\sum_{j=1}^{N} \frac{\partial \left[\nabla^2 \bar{\kappa}\right]^{-1}}{\partial g_j} \frac{dg_j}{d\tau_i} \right) \mathcal{E}, \tag{1}$$

where the impact of g_j on $\left[\nabla^2 \bar{\kappa}\right]^{-1}$ is given by $\frac{\partial\left[\nabla^2 \bar{\kappa}\right]^{-1}}{\partial g_j} = -\frac{1}{\eta} \frac{\tilde{\omega}_j^2}{g_j^2} H_j^{-1}$, and where

$$\mathcal{T} := \left(I - \left[\nabla^2 \bar{\kappa}\right]^{-1} \frac{\partial \mathcal{E}}{\partial \Delta} \right)^{-1}.$$

Back

Proposition

Let χ denote either μ_m, Σ_{mn}, or τ_i. Then the impact of a change in χ on the moments of log GDP are given by

$$\frac{d\,\mathrm{E}[y]}{d\chi} - \frac{\partial\,\mathrm{E}[y]}{\partial\chi} = \mu^\top \frac{d\Delta}{d\chi} \qquad \text{and} \qquad \frac{d\,\mathrm{V}[y]}{d\chi} - \frac{\partial\,\mathrm{V}[y]}{\partial\chi} = 2\Delta^\top\Sigma\frac{d\Delta}{d\chi},$$

where the use of a partial derivative indicates that Δ is kept fixed.

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Implications for GDP

Simplified model

Back

- Single risk factor $\varepsilon_t \sim$ iid $\mathcal{N}(0, \Sigma)$
- Firm level TFP is $\log TFP_{it} = \delta_{it}\varepsilon_t + \gamma_i t + v_{it}$ where γ_i is deterministic trend and $v_{it} \sim$ iid $\mathcal{N}(\mu_i^v, \Sigma_i^v)$

Implications for GDP

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Back

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Variance of firm-level TFP growth

$$\mathrm{V}\left[\log TFP_{it} - \log TFP_{it-1}\right] = 2\delta_i^2\Sigma + 2\Sigma_i^v$$

Simplified model

Back

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Covariance of firm-level TFP growth with GDP growth

$$\mathrm{Cov}\left[\log TFP_{it} - \log TFP_{it-1}, y_t - y_{t-1}\right] = 2\Delta\Sigma\delta_i + 2\tilde{\omega}_i\Sigma_i^v.$$

Implications for GDP

Simplified model

◂ Back

- Single risk factor $\varepsilon_t \sim$ iid $\mathcal{N}(0, \Sigma)$
- Firm level TFP is $\log TFP_{it} = \delta_{it}\varepsilon_t + \gamma_i t + v_{it}$ where γ_i is deterministic trend and $v_{it} \sim$ iid $\mathcal{N}(\mu_i^v, \Sigma_i^v)$

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$$\mathrm{Cov}\left[\log TFP_{it} - \log TFP_{it-1}, y_t - y_{t-1}\right] = 2\Delta\Sigma\delta_i + 2\tilde{\omega}_i\Sigma_i^v.$$

Model-implied firm risk exposure ($\mathcal{E} < 0$)

$$\delta_i = \delta_i^\circ + \frac{1}{\eta}\frac{\omega_i}{1+\tau_i}H_i^{-1}\mathcal{E}$$

$\Rightarrow$ Firms with large Domar weights and small markups are less volatile and less corr. with GDP

Markup estimation

◂ Back

- Assume Cobb-Douglas production function

$$\log Q_{it} = \alpha_{Li} \log L_{it} + \alpha_{Mi} \log M_{it} + \alpha_{Ki} \log K_{it} + \varepsilon_{it},$$

- Elasticities estimated using Levinsohn and Petrin (2003) with the Ackberg et al. (2015) correction.
    - Capital is the "state" variable, labor is the "free" variable and materials is the "proxy" variable.
- Production function estimated at NACE 2-digit sector level. As in De Loecker et al. (2020), we control for markups using firms' sales shares in the production function estimation.
- Following De Loecker and Warzynski (2012), we compute the markup as $1 + \tau_{it} = \hat{\alpha}_{Li} \Big/ \left(\frac{\text{Wage Bill}_{it}}{\text{Sales}_{it}} \right)$.
- We compute TFP growth as

$$\begin{aligned} \Delta \log \text{TFP}_{it} =& \Delta \log Q_{it} - \alpha_{Li} \Delta \log L_{it} - \alpha_{Mi} \Delta \log M_{it} - \alpha_{Ki} \Delta \log K_{it} \\ & - \left(\Delta \log \left(1 + \tau_{it}\right) - \Delta \log \left(1 + \tau_{s(i)t}\right) \right). \end{aligned}$$

The term $\Delta \log (1 + \tau_{it}) - \Delta \log \left(1 + \tau_{s(i)t}\right)$ accounts for the firm-specific markup growth net of the sectoral markup growth. This adjustment allows us to remove the change in firm-specific nominal price that are not taken into account by the sector-level price deflator.

TFP growth volatility

- We compute the standard deviation of TFP growth for each firm, $\sigma_i\left(\Delta \log TFP_{it}\right)$, and the time-series average of its markup and Domar weight.
- We construct deciles based on average Domar weights and markups, and create dummy variables, FE_{ji}^{Domar} and FE_{ji}^{Markup}, such that $FE_{ji}^{Domar} = 1$ if firm i's Domar weight is in decile j, and analogously for markups.
- We run the cross-sectional regression

$$\sigma_i\left(\Delta \log TFP_{it}\right) = \alpha + \sum_{j=1}^{10} \beta_j^{Domar} FE_{ji}^{Domar} + \sum_{j=1}^{10} \beta_j^{Markup} FE_{ji}^{Markup} + \varepsilon_i,$$

and plot β_j^{Domar} in panel (a) and β_j^{Markup} in panel (b).

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TFP growth volatility

- We construct deciles based on firms' Domar weights and markups each year.
- We then construct a set of dummy variables, FE_{jit}^{Domar} and FE_{jit}^{Markup}, such that $FE_{jit}^{Domar} = 1$ if firm i's Domar weight is in decile j in year t, and analogously for markups.
- We then run the following panel regression,

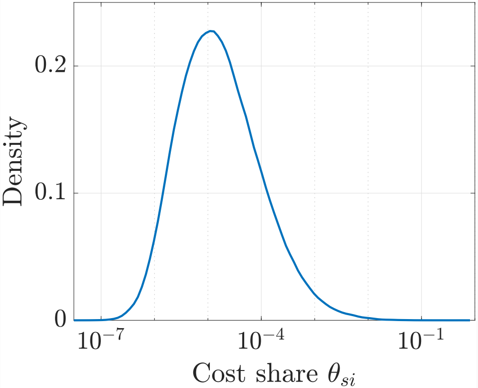
$$\Delta \log TFP_{it} = \sum_{j=1}^{10} \beta_j^{Domar} \left(FE_{jit}^{Domar} \times \Delta \log GDP_t \right) + \sum_{j=1}^{10} \beta_j^{Markup} \left(FE_{jit}^{Markup} \times \Delta \log GDP_t \right)$$
$$+ \alpha + \beta_0 \Delta \log GDP_t + \sum_{j=1}^{10} FE_{jit}^{Domar} + \sum_{j=1}^{10} FE_{jit}^{Markup} + \varepsilon_{it},$$

where $\Delta \log TFP_{it}$ is the annual growth of firm i's log TFP and $\Delta \log GDP_t$ is the annual growth of Spanish log GDP.

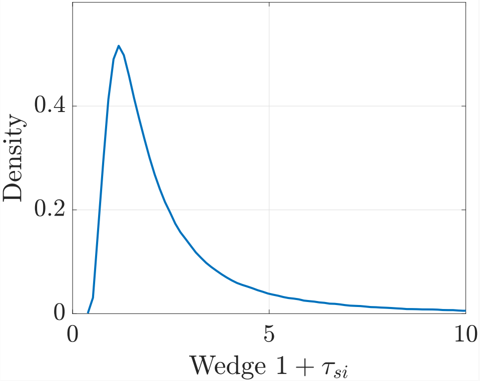
- The coefficients of interest, β_j^{Domar} and β_j^{Markup}, are reported in the figure.

◂ Back

Figure 1: Data distributions that the calibration matches exactly



(a) Sales share θ_{si}



(b) Wedges $1+\tau_i$

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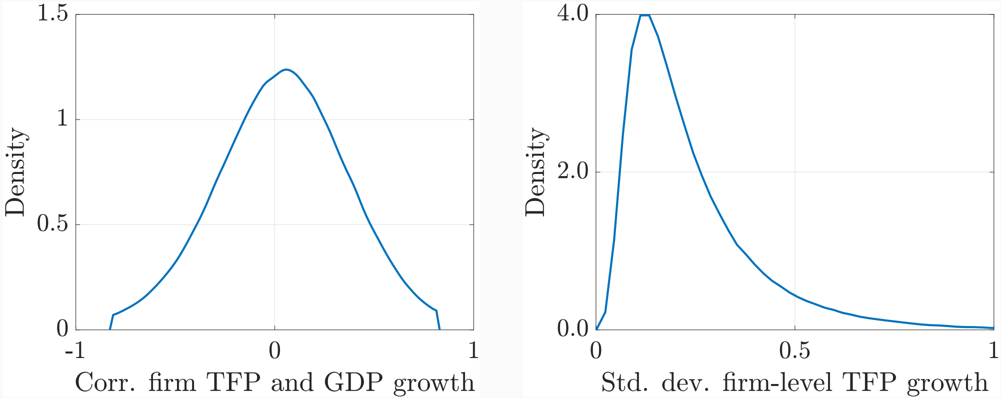
Calibrated model

Figure 2: Data distributions that the calibration matches exactly



(a) Correlation firm-level TFP and GDP growth **(b)** Standard deviation of firm-level TFP growth

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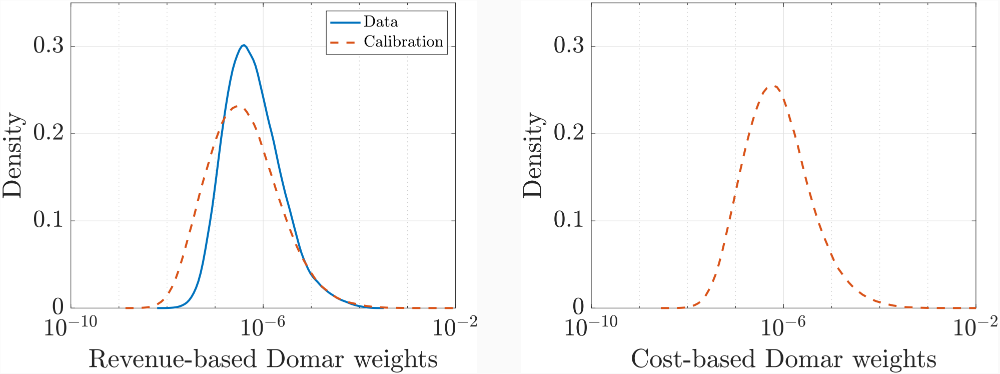
Calibrated model

Figure 3: Domar weights of the firms in the data and in the model



(a) Revenue-based Domar weights

(b) Cost-based Domar weights

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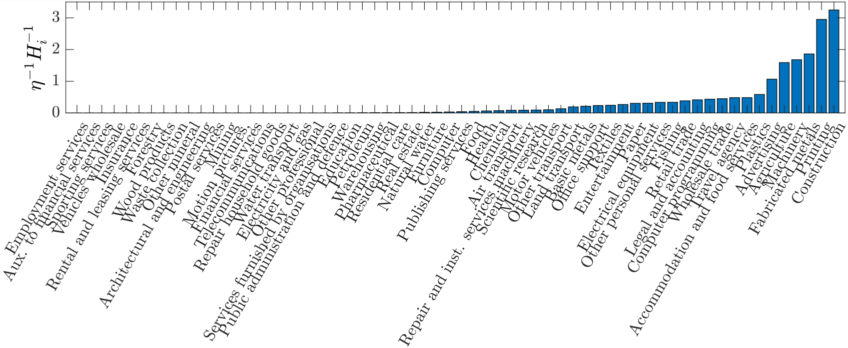
Figure 4: Estimated value of $\frac{1}{\eta}H_i^{-1}$ for each sector.



Notes. The scale of $\frac{1}{\eta}H_i^{-1}$ depends on our choice of ρ and Σ. We set $\rho = 5$ and $\Sigma = 1$ for this figure.

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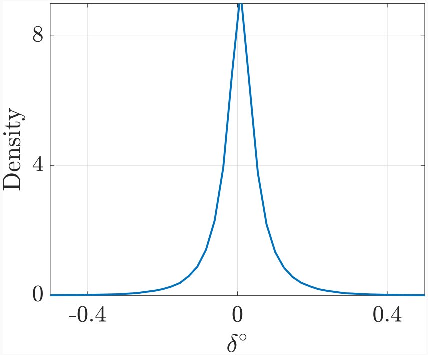
Figure 5: Distribution of the estimated firm-level natural risk exposure δ_i° /



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